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OCTOBER, 1860.

EDITED BY

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Editorial Items.

Prizes for Vol. III. offered to all Students.

For Solutions. — A 1st Prize, $7.50; 2d, $5; 3d, $3; 4th, Vol. III. The third and fourth prizes are confined to beginners for solutions of Problems I. and II.; but as the best solutions only will be published, advanced students are desired to send solutions of all the problems.

For Essays. — A 1st Prize, $50; 2d, $40; 3d, $30; 4th, $20; 5th, $10. The first and second of these prizes are open to all competitors. Instead of cash prizes, we propose to make them all payable in mathematical books. This change will make the prizes less burdensome to us while it will be appropriate, and we believe quite as acceptable to competitors, to receive standard mathematical books, — such as Bowditch's edition of *Mécanique Céleste*, Peirce's *Analytic Mechanics*, Bartlett's *Analytic Mechanics*, Davis's edition of *Theoria Motus*, Davies and Peck's *Mathematical Dictionary*, Chauvenet's *Analytic Trigonometry*. But selections may be made from Sever & Francis's list, and desired works not found there will be ordered. To those preferring standard works in other departments, we offer Worcester's *Quarto Dictionary* and the fine editions of Hume, Gibbon, Lamb, and Macaulay, described in our advertising pages. To this list we may hereafter add editions of other standard authors, if desired.

One Plane Descriptive Geometry.

Mr. Watson writes us that his work on this subject (about fifty pages of text and twelve plates) is nearly ready. The plates will be engraved in Paris by the same engraver, and printed on the same quality of paper, as those in Leroy's *Descriptive Geometry*. A supplement will contain an application to a road constructed in France by the Corps des Ponts et Chaussées.

Autograph Note of Laplace.

Mr. Watson has sent us lithograph copies of a note in the handwriting of the distinguished Geometer, which will appear in our next.

Portrait of Sir John Herschel.

Through the kindness of Miss Maria Mitchell we have received a fine photograph of Sir John Herschel, which we shall have engraved on steel for an early number.

We expect our friends will aid us by increasing our subscription list. See Mathematical Monthly, on seventh page of Advertiser.

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THE

MATHEMATICAL MONTHLY.

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PRIZE PROBLEMS FOR STUDENTS.

I. Let AB and CD be two diameters of a given circle, drawn at right angles to each other; AEB a circular arc described with radius DA or DB. Prove that the area of the lune $AEBC$ equals the area of the triangle ADB.

II. Of all isoperimetrical polygons having the same number of sides, the greatest is that which is equilateral.

III. In any triangle, if P, P', and P'' denote the perpendiculars from the vertices to the opposite sides; p, p', and p'' the portions of the same between their point of intersection and the sides. Prove that

$$\frac{p}{P} + \frac{p'}{P'} + \frac{p''}{P''} = 1.$$

— Communicated by J. A. WINEBRENER, Princeton College, N. J.

IV. At what angle must the rudder of a vessel be inclined to the stream, that the effect produced may be a maximum?

V. Two circles, radii r and $r_{,}$, have their centres united by a line d feet long. Required the position upon this line of the centre of a third circle, which shall cut off a square feet from the first circle and b feet from the second. — Communicated by ASHER B. EVANS.

Solutions of these problems must be received by December 1, 1860. (See Editorial Items.)

NOTE* ON SOME PROPERTIES OF SQUARE NUMBERS.

By Prof. D. W. Hoyt, Fairfax, Vt.

If all *differences* of square numbers which produce squares are known, the *sums* of squares which produce squares are also known; for the one is obtained from the other by the transposition of a single term of the equation.

The differences between consecutive squares of the series 0, 1, 4, 9, 16, 25, &c., are 1, 3, 5, 7, 9, &c., a series including all the odd numbers, and consequently the squares of all odd numbers. The square of every odd number is, therefore, equal to the difference between the squares of two numbers differing by unity. 1 is the difference between 0 and 1; the second odd square is 9, which is the difference between 16 and 25; the third is 25, which is the difference between 144 and 169; that is, $1^2 = 1^2 - 0^2$, $3^2 = 5^2 - 4^2$, and $5^2 = 13^2 - 12^2$, from which other equations of the series can readily be obtained, as the common difference of the first members is 2, and the terms of the second members have 4 as a common difference of the second order.

The differences between alternate terms of the series of square numbers are 4, 8, 12, 16, &c., including all numbers divisible by 4; hence the square of every even number is equal to the difference between the squares of two numbers differing by 2. By an inspection of these series, we obtain $2^2 = 2^2 - 0^2$, $4^2 = 5^2 - 3^2$, $6^2 = 10^2 - 8^2$, &c.

The differences between the squares of numbers differing by 3 include all odd numbers divisible by 3, and consequently all those divisible by 9; hence the square of every odd number divisible by 3

* This note was furnished by Prof. Hoyt, in connection with that published on pp. 264, 265, May, 1860, but was then omitted for want of space. — Ed.

is equal to the difference between the squares of two numbers differing by 3. The differences between the squares of numbers differing by 4 include all numbers divisible by 8, and consequently all those divisible by 16, or the squares of those divisible by 4. The use of numbers differing by 5, 6, and 7, as well as 3 and 4, give equations which are multiples of those contained in the first two series; but 8 and 9 introduce new relations of numbers. The differences between the squares of numbers differing by 8, in the series $(-3)^2$, $(-2)^2$, $(-1)^2$, 0^2, 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, &c., are 16, 32, 48, 64, 80, &c., including all numbers divisible by 16; and consequently the squares of all numbers divisible by 4, giving a series of equations whose first members are the same as when the numbers differ by 4. The square of every odd number divisible by 3 is also equal to the difference between the squares of two numbers differing by 9, as well as two others differing by 3.

In general, if m represent any odd number whatever, then the square of every odd number divisible by m will be equal to the difference between the squares of two numbers differing by m. If n represent any even number, the square of every number divisible by n will be equal to the difference between the squares of two numbers differing by n. If m contains a square factor, or if n contains a square factor in addition to the factor 2, there will also be numbers not divisible by m or n whose squares will be equal to the differences between the squares of numbers differing by m or n. The following statement, however, includes all possible cases: Every odd square divisible by m will be equal to the difference between the squares of two numbers differing by m, and every square divisible by $2n$ will be equal to the difference between the squares of two numbers differing by n.

Since $18 = 2 \times 3^2$ and $25 = 5^2$, the use of those numbers as differences will introduce some equations not multiples of any preceding ones.

The same results may be more readily and satisfactorily reached by another process. The general equation is of the form $M^2 = N^2 - P^2$. Let $N - P = p$, or $P = N - p$, and the general equation becomes $M^2 = 2pN - p^2$. As this equation contains three variable quantities, we may give any values whatever to two of them, and the third will be a function of those two. But if we consider either M or p as the function, the equation will assume the quadratic form, and we cannot attribute all values to the arguments without producing surds. It will therefore be best to consider M and p the arguments, and N the function, or $N = \frac{M^2 + p^2}{2p}$. If we attribute any rational values whatever to M and p, the values of N and $N - p$ will be rational; but our present purpose is to deal only with whole numbers, and we can, therefore, only substitute such integers for M and p as shall render N a whole number. It is evident that any whole number may be used for p, and that M^2 must in all cases be divisible by p. Since $N = \frac{1}{2}\left(\frac{M^2}{p} + p\right)$, $\frac{M^2}{p} + p$ must be an even number; consequently if p is an odd number, $\frac{M^2}{p}$ must be odd, that is, M^2 may be any odd square divisible by p; and if p is an even number, $\frac{M^2}{p}$ must be even, that is, M^2 may be any square divisible by $2p$.

If we make $p = 1$, and regard it as constant, M may be any odd number whatever. When $p = 2$, M^2 must be divisible by 4, and M may be any even number whatever. If $p = 3$, M may be any odd number divisible by 3, and so on. For each value of p we have an infinite series of equations; and since p may have an infinite number of values, we have an infinite number of infinite series. As p is constant for any given series, N and $N - p$ may be readily found by the above formula for any possible value of M. When three equations are thus obtained, the law of the series becomes evident, and the other equations may be obtained without the aid of the formula.

It is worthy of notice that each series, in descending, returns upon itself, — except that M becomes negative, — and each series is thus unlimited in both directions. If the series where $p = 8$ be continued backwards, we have $0^2 = 4^2 - (-4)^2$, $(-4)^2 = 5^2 - (-3)^2$, $(-8)^2 = 8^2 - 0^2$, $(-12)^2 = 13^2 - 5^2$, &c. The formula also shows that M may be made either positive or negative without any change in the other quantities.

As $M^2 + p^2$ must always be positive, it is evident that if we make p negative, N must also be negative (M being either positive or negative, as before), and the series will differ from those produced by the corresponding positive values of p only in having N and $N - p$ take an opposite sign. When $p = 0$, $N = \infty$, and $M^2 = \infty^2 - \infty^2$, unless M is also 0, when $0^2 = N^2 - N^2$. In the two equations last named, M and N represent any finite quantity whatever.

The following table contains the most interesting and important of the first thirty series. Others, up to the thirty-second (where $p = 32$), are merely multiples of those given.

$p = m = 1.$

$1^2 = 1^2 - 0^2$
$3^2 = 5^2 - 4^2$
$5^2 = 13^2 - 12^2$
$7^2 = 25^2 - 24^2$
$9^2 = 41^2 - 40^2$
$11^2 = 61^2 - 60^2$
$13^2 = 85^2 - 84^2$

$p = n = 2.$

$2^2 = 2^2 - 0^2$
$4^2 = 5^2 - 3^2$
$6^2 = 10^2 - 8^2$
$8^2 = 17^2 - 15^2$
$10^2 = 26^2 - 24^2$
$12^2 = 37^2 - 35^2$
$14^2 = 50^2 - 48^2$
$16^2 = 65^2 - 63^2$
$18^2 = 82^2 - 80^2$

$p = n = 8.$

$4^2 = 5^2 - (-3)^2$
$8^2 = 8^2 - 0^2$
$12^2 = 13^2 - 5^2$
$16^2 = 20^2 - 12^2$
$20^2 = 29^2 - 21^2$
$24^2 = 40^2 - 32^2$
$28^2 = 53^2 - 45^2$
$32^2 = 68^2 - 60^2$
$36^2 = 85^2 - 77^2$

$p = m = 9.$

$3^2 = 5^2 - (-4)^2$
$9^2 = 9^2 - 0^2$
$15^2 = 17^2 - 8^2$
$21^2 = 29^2 - 20^2$
$27^2 = 45^2 - 36^2$
$33^2 = 65^2 - 56^2$
$39^2 = 89^2 - 80^2$

$p = n = 18.$

$6^2 = 10^2 - (-8)^2$
$12^2 = 13^2 - (-5)^2$
$18^2 = 18^2 - 0^2$
$24^2 = 25^2 - 7^2$
$30^2 = 34^2 - 16^2$
$36^2 = 45^2 - 27^2$
$42^2 = 58^2 - 40^2$
$48^2 = 73^2 - 55^2$
$54^2 = 90^2 - 72^2$

$p = m = 25.$

$5^2 = 13^2 - (-12)^2$
$15^2 = 17^2 - (-8)^2$
$25^2 = 25^2 - 0^2$
$35^2 = 37^2 - 12^2$
$45^2 = 53^2 - 28^2$
$55^2 = 73^2 - 48^2$
$65^2 = 97^2 - 72^2$

NOTE ON THE "HORIZONTAL THRUST OF EMBANKMENTS."

By Prof. F. W. Bardwell, Antioch College, Yellow Springs, Ohio.

I have taken the earliest opportunity since my return to examine more carefully the question of the horizontal thrust of an embankment of earth rising from the retaining wall in a natural slope, and I am happy to find that my former criticism upon an article by Captain Woodbury was entirely at fault, and that his formula was correct.

I think I understood in the first place the conditions of the problem, but was misled by relying, without sufficient examination, upon a statement in the American edition of Weisbach's Mechanics. The question of the horizontal thrust of an embankment terminating in a horizontal plane is there discussed at length, and the usual formula deduced. But again (Vol. II., page 18, *Remark*) it is stated that this same formula is applicable to the case where the earth is carried above the wall in a natural slope, provided the height of the earth be substituted in the formula for that of the wall. At first this seemed plausible; and, accepting it without further scrutiny, I was afterwards led to criticise too hastily the article of Captain Woodbury.

The formula given by Weisbach is,

$$P = \tfrac{1}{2}\, h^2 \gamma \tan^2 \left(45° - \tfrac{\varrho}{2}\right);$$

P being the horizontal thrust, h the height of the wall, γ the density of the earth, and ϱ the angle of friction. To show that this formula is not applicable in the case of a sloping surface, by making h equal to the height of the earth, let us suppose the simple case where the height of the wall is zero, and the earth extends back indefinitely with its natural slope. Now the horizontal thrust is obviously zero, but the above formula would make it indefinitely great.

I have quoted from the American edition of WEISBACH, which contains some additions to the original; but I have not a copy of the German edition at hand to consult with reference to this point.

Let me add, with regard to the paper of Captain WOODBURY, that a careful examination shows the elegance of the method he used, and the simplicity of the formula deduced for its practical application.

NOTE ON THE DEPENDENCE OF EQUATIONS.

By JOHN BORDON, Chicago, Illinois.

A METHOD of deducing equations of a higher order from those of a less degree, by a method of combining the imaginary roots of $x^3 - a^3$, $x^5 - a^5$, &c., has been heretofore shown. The results thereby arrived at can be more easily obtained in some cases by the law of symmetrical functions. For, suppose we have an equation, $x^2 + b\,x + c = 0$, and we find the coefficients of another equation of the same degree whose roots shall be the cubes of those in the above equation; and suppose $x_{,}^2 + B\,x_{,} + C = 0$ to be the equation; then if $-a_{,}, -b_{,}$, are the roots of the first equation, then $B = a_{,}^3 + b_{,}^3$, $C = a_{,}^3\ b_{,}^3$. But by symmetrical functions, if we represent by S_1, S_2, S_3, &c., the sum of the first power, sum of the squares, sum of the cubes, &c., of the roots of the first equation, we have

$$S_1 = -b\,;\ S_2 = b^2 - 2\,c\,;\ S_3 = -b^3 + 3\,b\,c.$$

But $B = -S_3 = +b^3 - 3\,b\,c$, and since $c = a_{,}\,b_{,}$, evidently $C = a_{,}^3\,b_{,}^3$, or $c = C^{\frac{1}{3}}$. Hence we have

$$b^3 - 3\,C^{\frac{1}{3}}\,b - B = 0.$$

Here we have an equation of the third degree deduced out of two others of the second degree, and by this means we can arrive at

the CARDAN formula. For if the above cubic be the same as $x_{''}^3 + p\,x_{''} + q = 0$, then $x_{''} = b$, $p = -3\,C^{\frac{1}{3}}$, $q = -B$, or $C = -\frac{p^3}{27}$, $B = -q$, and we have

$$x_{'}^2 - q\,x - \frac{p^3}{27} = 0, \quad \text{or} \quad x_{'} = +\frac{q}{2} \pm \sqrt{\frac{q^2}{4} + \frac{p^3}{27}};$$

but $x_{'} = x^3$, or $x = x_{'}^{\frac{1}{3}}$, and $b =$ sum of the two values of x taken with contrary signs; and as

$$-x = \left(-\frac{q}{2} \pm \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}\right)^{\frac{1}{3}},$$

then

$$x_{''} = b = \left(-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}\right)^{\frac{1}{3}} + \left(-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}\right)^{\frac{1}{3}}.$$

So, if we went on obtaining the values of S_4, S_5, S_6, S_7, we would obtain equations of the 4th, 5th, 6th, and 7th degrees, which would be resolvable in the same manner. These would give the forms of equations of those degrees which depend for their solution upon the quadratic. The two modes of solving biquadratics can be arrived at also by symmetrical functions, but the manner of doing so, heretofore pointed out, cannot be simplified. When we arrive at an equation of the 5th degree of the general form $x^5 + p\,x^3 +$ &c., the mode of solving cubics and biquadratics by the use of two equations of the next lower degree becomes more difficult of application. But the equation $x^5 + p\,x^3 +$ &c. can be deduced from two equations of the fourth degree. And it yet remains to be seen whether a solution of the quintic cannot be fairly looked for in that direction. And the general proposition may be stated, that an equation of the form $x^m + p\,x^{m-2} +$ &c. $= 0$ may be derived from two equations of the form

(1) $$x_{'}^{m-1} + A\,x_{'}^{m-2} + \text{&c} = 0,$$

(2) $$x_{''}^{m-1} + a\,x_{''}^{m-2} + \text{&c} = 0;$$

which shall stand to each other in the relation that the roots of Eq. (1) are the mth power of the roots of Eq. (2), i. e. $x_{,,}^{\,m} = x_{,}$.

And to illustrate this, let us deduce $x^5 + px^3 +$ &c. from two biquadratics. First, we have

$$x_{,}^{\,4} + Ax_{,}^{\,3} + Bx_{,}^{\,2} + Cx_{,} + D = 0, \tag{3}$$

$$x_{,,}^{\,4} + ax_{,,}^{\,3} + bx_{,,}^{\,2} + cx_{,,} + d = 0; \tag{4}$$

in which we suppose $x_{,} = x_{,,}^{\,5}$. Now let us arrive at the value of A in terms of a by symmetrical functions: —

$$S_1 = -a\,;\ S_2 = a^2 - 2b\,;\ S_3 = -a^3 + 3ab - 3c\,;$$

$$S_4 = a^4 - 4a^2b + 4ac + 2b^2 - 4d\,;$$

$$S_5 = -a^5 + 5a^3b - 5a^2c - (5b^2 - 5d)\,a + 5bc.$$

But $$A = -S_5 = a^5 - 5a^3b +$$ &c.

Hence

$$a^5 - 5ba^3 + 5ca^2 + (5b^2 - 5d)\,a - 5bc - A = 0. \tag{6}$$

Hence, if we have

$$x^5 + px^3 + qx^2 + rx + s = 0, \tag{7}$$

then $a = x$; $p = -5b$; $q = 5c$; $r = 5b^2 - 5d$; $S = -A - 5bc$; that is to say, a root of an equation of the 5th degree, as Eq. (7), will be equal to the sum of the roots of an equation of the fourth degree, as Eq. (4), taken with a contrary sign, when $b = -\frac{p}{5}$; $c = \frac{q}{5}$; $d = \frac{p^2 - 5r}{25}$; $A = \frac{pq}{5} - S$. Now D in Eq. (3) is equal to d^5, hence $D = \left(\frac{p^2 - 5r}{25}\right)^5$; and if B and C were determined in like manner, then x in Eq. (7) would be equal to the sum of the 5th roots of Eq. (3) taken with their signs changed, and the quintic would be resolved into the biquadratic. The law of symmetrical functions renders the general proposition obvious.

NOTE ON NUMERICAL EQUATIONS.

By Prof. A. Vallas, State Seminary and Military Academy, Alexandria, Louisiana.

The Resolution of Numerical Equations, so far at least as real roots are concerned, has been brought to such perfection that scarcely more is expected at present than to render approximations more easy and expeditious.

It is known that the common method of finding the irrational real roots of equations is still that of Newton, although considerably amended and perfected by such men as Budan, Fourier, and Horner.

Geometrically this common method consists in this. The equation being referred to two rectangular axes, and the real roots being the abscissas of the intersections of the horizontal axis by the curve, from an ordinate and trigonometrical tangent of a point near an intersection, to find this intersection by means of the "subtangent."

For in the equation

$$fx = x^m + Ax^{m-1} + Bx^{m-2} + \ldots + Kx + L = 0,$$

put $a + x'$ for x, the results will be,

$$f(a+x') = fa + f'a \,.\, x' + \frac{f''a}{1\,.\,2}x'^2 + \ldots + \frac{f^{(m)}a}{1\,.\,2\,\ldots\,m}x'^m = 0.$$

If now a be near one of the real roots, we get, in general, a still nearer value by putting

$$fa + f'a \,.\, x' = 0\,,$$

from which

$$x' = -\frac{fa}{f'a}\,;$$

and the nearer value will be

$$a + x' = a - \frac{fa}{f'a}.$$

That here x' is the subtangent of the point whose abscissa is a, needs not to be proved.

How that, in order to a sure approximation, the root ought first to be included between such limits as contain no singular points, as also how the degree of approximation to be obtained may be determined beforehand, has, with other particulars, been admirably discussed by FOURIER; although it must be regretted that FOURIER despised to connect his disquisitions with those of BUDAN, whose transformed equations (or equations where roots are all less by a) afford very easy means for determining the numerical values of

$$fa,\ f'a,\ f''a,\ f'''a \ldots,$$

which form the substratum of FOURIER's discussion. The two proceedings were at last connected by others, and a method engrafted on this connection, namely, also by HORNER, which may be regarded as the triumph of modern investigation in these matters.

It were needless here to do more than merely mention, that the diminution of roots is, geometrically, equivalent to transposing the axis of the ordinates to the right hand, and that according to this the approximations are obtained by subtangents belonging to points of the curve situated in that axis.

In this note it is proposed to show: 1st, that the approximations aimed at may be more easily obtained by abscissas of parabolic arcs tangent to the curve where it intersects the axis of the ordinates; and, 2d, to give a sure rule (as FOURIER has done for NEWTON's method) for the degree of approximation thus obtained.

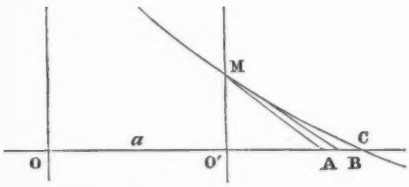
In regard to the first point, let

$$f(a+x') = fa + f'a \,.\, x' + \frac{f''a}{1.2} \,.\, x'^2 + \ldots = 0$$

be such a transformed equation as excludes all singular points between the point belonging to a and the nearest intersection, and put

$$fa + f'a \,.\, x' + \frac{f'a}{1.2} \,.\, x'^2 = 0;$$

then will this latter equation manifestly belong to a parabolic curve, having a contact of the second order, at the point belonging to a, with the curve represented by the equation $fx = 0$, or having that contact in the axis of the ordinates, as this axis is supposed to have been removed by as much as a to the right hand.



In the annexed figure, O being the origin of the co-ordinates of the curve represented by $y = fx$, O' will be that of the curve $y' = f(a + x')$. Further, M being the point of the curve in the new axis of the co-ordinates, $O'A$ is the approximation of the nearest intersection by the subtangent, while $O'B$ is that by the abscissa of the parabolic arc in question.

Solving the equation

$$fa + f'a \,.\, x' + \frac{f''a}{2} x'^2 = 0,$$

we obtain $$x' = -\frac{f'a}{f''a} \pm \frac{f'a}{f''a} \sqrt{1 - \frac{2fa \,.\, f''a}{(f'a)^2}},$$

in which expression the upper sign is to be taken; for, 1st, if x' admits values of different signs, the conditions of the problem exclude the negative one; and, 2d, if both should be positive, those same conditions require us to take the less.

Developing now the expression for x', just found, we get

$$x' = -\frac{fa}{f'a} - \left(\frac{fa}{f'a}\right)^2 \cdot \frac{f''a}{2f'a} - 2\left(\frac{fa}{f'a}\right)^3 \cdot \left(\frac{f''a}{2f'a}\right)^2 \ldots$$

This is the expression for the abscissa of the parabolic arc, or for $O'B$, which abscissa is evidently made up of the subtangent $O'A$ represented by the first, and the portion AB represented by the succeeding terms of the development.

Now, in regard to the degree of approximation, it is evident that in developing in a similar way the value of x'' from

$$f a + f' a \,.\, x'' + \frac{f''(a+\theta x'')}{2} \,.\, x''^2 = 0,$$

which is manifestly the given equation itself; we obtain for $x'' = O'C$,

$$O'C = -\frac{f a}{f' a} - \left(\frac{f a}{f' a}\right)^2 \,.\, \frac{f''(a+\theta x'')}{2 f' a} - 2\left(\frac{f a}{f' a}\right)^3 \,.\, \left(\frac{f''(a+\theta x'')}{2 f' a}\right)^2 \ldots$$

Or, developing $f''(a + \theta x'')$,

$$O'C = -\frac{f a}{f' a} - \left(\frac{f a}{f' a}\right)^2 \,.\, \frac{f'' a}{2 f' a} - 2\left(\frac{f a}{f' a}\right)^3 \,.\, \left(\frac{f'' a}{2 f' a}\right)^2 \ldots$$
$$- \theta x'' \,.\, \left(\frac{f a}{f' a}\right)^2 \,.\, \frac{f''' a}{2 f' a} \ldots$$

Leaving out the other terms, we see easily that the difference of the two abscissas $O'C - O'B = BC$ is

$$= -\theta x'' \left(\frac{f a}{f' a}\right)^2 \,.\, \frac{f''' a}{2 f' a},$$

$$= -3\theta x'' \,.\, \left(\frac{f a}{f' a}\right)^2 \,.\, \frac{f''' a}{2 \,.\, 3 f' a};$$

and this expression we shall directly discuss.

Suppose the transformed equation to be

$$x'^m + B_{m-1} x'^{m-1} + B_{m-2} x'^{m-2} + \ldots + B_3 x'^3 + B_2 x'^2 + B_1 x' + B_0 = 0;$$

then we shall have

$$B_0 = f a, \quad B_1 = f' a, \quad B_2 = \frac{f'' a}{2}, \quad B_3 = \frac{f''' a}{2 \,.\, 3} \ldots$$

Substituting these values, the difference between the abscissa of the curve represented by the given equation and that of the parabolic arc, or

$$BC \text{ will be } = -3\theta x'' \left(\frac{B_0}{B_1}\right)^2 \,.\, \frac{B_3}{B_1}.$$

If now $$\frac{B_3}{B_1} = \frac{1}{10^m},$$

and we suppose a to be a portion of the root with n correct decimals, that is, x'', and (which is very nearly the same)

$$\frac{B_0}{B_1} = \frac{1}{10^n};$$

we see easily that the difference $O'C - O'B = BC$ will be numerically

$$= \frac{3}{10^{m+3n}},$$

and we get therefore, in all cases, through the abscissa of the parabolic arc, at least $m + 3n - 1$ correct decimal places.

Let us now return for a moment to the development of this abscissa.

$$O'C = -\frac{fa}{f'a} - \left(\frac{fa}{f'a}\right)^2 \cdot \frac{f''a}{2f'a} - 2\left(\frac{fa}{f'a}\right)^3 \cdot \left(\frac{f''a}{2f'a}\right)^2 \cdots$$

By a similar substitution, the third term becomes

$$-2\left(\frac{B_0}{B_1}\right)^3 \cdot \left(\frac{B_2}{B_1}\right)^2.$$

If we now put $$\frac{B_2}{B_1} = \frac{1}{10^\mu},$$

we shall have $$2\left(\frac{B_0}{B_1}\right)^3 \cdot \left(\frac{B_2}{B_1}\right)^2 = \frac{2}{10^{2\mu+3n}};$$

hence this term can only affect the $(2\mu + 3n)$th decimal, and may therefore be neglected, when even

$$2\mu \leq m - 1.$$

Having therefore, by diminutions of the roots, sufficiently approached one of the irrational real roots, and having thus obtained a transformed equation of the form

$$x'^m + B_{m-1}x'^{m-1} + \ldots + B_3 x'^3 + B_2 x'^2 + B_1 x' + B_0 = 0,$$

we shall find $x' = -\frac{B_0}{B_1} - \left(\frac{B_0}{B_1}\right)^2 \cdot \frac{B_2}{B_1} - 2\left(\frac{B_0}{B_1}\right)^3 \cdot \left(\frac{B_2}{B_1}\right)^2$

for an approximation correct in $m + 3n - 1$ decimals, if by previous operations n have been found already. And in this it must be remarked, 1st, that m is to be determined by the relation,

$$\frac{B_3}{B_1} \leqq \frac{1}{10^m};$$

and that, 2d, the third term of the value of x' may be omitted whenever $2\mu \leqq m$, μ being determined by the relation,

$$\frac{B_2}{B_1} < \frac{1}{10^\mu}.$$

Let us take, for an example, the equation

$$x^3 - 10x^2 - 130x + 858 = 0.$$

It is easy to find that the real roots are contained between 5 and 6, 14 and 15, —10 and —11. In order to find the first of these roots, let us diminish all roots by 5.

5)1	— 10	— 130	+ 850
1 —	5	— 155	+ 75
1	0	— 155	

$$x'^3 + 5x'^2 - 155x' + 75 = 0.$$

It is easy to find the first decimal. According to the common method, there is sometimes a doubt about this. According to our proceeding,

$$-\frac{B_0}{B_1} = \frac{75}{155} = 0.48; \qquad -\left(\frac{B_2}{B_1}\right)^2 \cdot \frac{B_2}{B_1} = \left(\frac{75}{155}\right)^2 \cdot \frac{5}{155} = 0.00;$$

and there is therefore no doubt about this first decimal. Diminishing by 0.4,

0.4)1	5	— 155	75
1	5.4	— 152.84	13.864
1	5.8	— 150.52	

$$x''^3 + 6.2x''^2 - 150.52x'' + 13.864 = 0.$$

Before proceeding further, let us examine how many decimals we shall get by the application of our proceeding.

$$\frac{B_3}{B_1} = \frac{1}{150} < \frac{1}{10^2}, \qquad \frac{B_2}{B_1} = \frac{6}{150} < \frac{1}{10}.$$

We have, therefore, $m = 2$, $\mu = 1$. This will lead us to the fourth decimal inclusively. We shall have

$$-\frac{B_0}{B_1} = 0.09210$$
$$-\left(\frac{B_0}{B_1}\right)^2 \cdot \frac{B_2}{B_1} = \underline{0.00034}$$
$$0.0924$$

The next approximation would give us thirteen correct decimals. If we diminish, however, only by 0.09, we shall obtain at least seven, and this we shall do.

0.09) 1	6.2	— 150.52	13.864
	9	0.5661	— 13.495851
	6.29	— 149.9539	0.368149
	9	0.5742	
	6.38	— 149.3797	
	9		

$$x'''^3 + 6.47\,x'''^2 - 149.3797\,x''' + 0.368149 = 0;$$

from which

$$-\frac{B_0}{B_1} = 0.00246\ 451$$
$$-\left(\frac{B_0}{B_1}\right)^2 \cdot \frac{B_2}{B_1} = \underline{0.00000\ 026}$$
$$0.00246\ 478$$

The root with seven correct decimals is, therefore, 5.4924647.

In applying this method to binomial equations, $x^n + N = 0$, put $x' + a$, instead of x:

$$x'^n + n\,a\,x'^{n-1} + \frac{n(n-1)}{1 \cdot 2}\,a^2\,x'^{n-2} + \ldots$$
$$+ \frac{n(n-1)}{1 \cdot 2}\,a^{n-1}\,x' + (a^n + N) = 0.$$

As here $B_0 = a^n + N$, $B_1 = n\,a^{n-1}$, $B_2 = \frac{n\,(n-1)}{1\,.\,2}\,a^{n-2}\ldots$, we get as an approximate value,

$$x' = -\frac{1}{n}\left(a + \frac{N}{a^{n-1}}\right) - \frac{(n-1)}{2\,a}\left[\frac{1}{n}\left(a + \frac{N}{a^{n-1}}\right)\right]^2 - \frac{(n-1)^2}{2\,a^2}\left[\frac{1}{n}\left(a + \frac{N}{a^{n-1}}\right)\right]^3 \ldots$$

And we get again at least $m + 3\,n' - 1$ correct decimals, after n' have been found, m being determined by the relation

$$\frac{(n-1)\,(n-2)}{1\,.\,2} \cdot \frac{1}{a} = \frac{1}{10^m}.$$

If $n = 2$, we obtain the following for the extraction of the square root from the number $-N$:

$$x' = -\tfrac{1}{2}\left(a + \frac{N}{a}\right) - \frac{1}{8\,a}\left(a + \frac{N}{a}\right)^2 - \frac{1}{16\,a}\left(a + \frac{N}{a}\right)^3 \ldots$$

In this case, the approximation is unlimited, as $\frac{(n-1)\,(n-2)}{2\,.\,3} = 0$; that is $m = \infty$.

If, for instance, $N = -12$, and $a = 3.46$, we get from the first three terms,

$$-\tfrac{1}{2}\left(a + \frac{N}{a}\right) = +0.00410\ 40462\ 4$$
$$-\frac{1}{8\,a}\left(a + \frac{N}{a}\right)^2 = -\qquad 24339\ 8$$
$$-\frac{1}{16\,a}\left(a + \frac{N}{a}\right)^8 = +\qquad 28\ 8$$
$$x' = +0.00410\ 16151\ (4);$$

hence the square root of 12 is 3.46410 16151.

It may be remarked here, that the first term of the development of x', namely, $-\frac{1}{n}\left(a + \frac{N}{a^{n-1}}\right)$, leads to an old rule, according to which, in the extraction of the nth root, nearly twice as many figures are to be found as are contained in a, and which rule is the follow-

ing: "Divide the number by the $(n-1)$st power of the part found (a), subtract the quotient from a, and divide the difference by n." It may also be remarked, that the same first term, as applied to the extraction of the square root, gives this root in nearly twice as many figures by the formula,

$$-\frac{a^2+N}{2a} = -\frac{a^2-N'}{2a} = \frac{N'-a^2}{2a},$$

(as N is supposed to be essentially negative,) which is nothing but a well-known rule, although commonly overlooked.

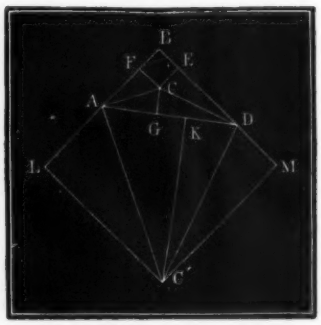
REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN No. IX., Vol. II.

The first Prize is awarded to Asher B. Evans, Madison Univ., N. Y.
The second prize is awarded to O. B. Wheeler, Michigan University.
The third prize is awarded to J. B. Davison, Oberlin College, Ohio.

Prize Solution of Problem I.

By Miss Harriet S. Hazeltine, Worcester, Mass.

I. The sum of the radii of the inscribed circle and of the circle touching the hypothenuse and extension of the two legs of a right triangle is equal to the sum of the said legs.



Find C and C', the centres of the two circles, by bisecting the angles including the hypothenuse AD, and their supplements. Then CE, CF, CG, perpendiculars to BD, AB, AD, will be radii of the inscribed circle, and $C'K$, $C'L$, $C'M$, perpendiculars to AD and to AB and BD prolonged, will be radii of the circle

whose centre is C'. $C'MBL$ is a square; $\therefore$ radius $C'M = AB + AL = AB + AK = AB + ED$ (Vol. II., No. VI., Problem II.). $CEBF$ is also a square; $\therefore$ radius $CE = EB$, and

$$C'M + CE = AB + ED + EB = AB + BD.$$

Prize Solution of Problem II.

By Miss Harriet S. Hazeltine, Worcester, Mass.

II. The side of a regular octagon, inscribed in a circle whose radius is unity, is $\sqrt{(2-\sqrt{2})}$, and its area is $2\sqrt{2}$. Required the proof.

Denote by AB the side of the inscribed square, and draw the radii CA, CB to its extremities. Draw also the radius CDE perpendicular to and bisecting AB at D, and the subtended arc at E. The chord AE is the side of the regular inscribed octagon. $AB = \sqrt{2}$; $AD = \frac{1}{2}AB = \frac{1}{2}\sqrt{2}$. The angle $CAD = ACD \therefore CD = AD = \frac{1}{2}\sqrt{2}$. $DE = CE - CD = 1 - \frac{1}{2}\sqrt{2}$. $AE = \sqrt{(\overline{AD}^2 + \overline{DE}^2)} = \sqrt{(2-\sqrt{2})}$. Four times the area of the triangle $ABE = 4 \times AD \times DE = 2\sqrt{2} - 2$. Adding this to the area of the inscribed square, which is 2, we shall obtain the area of the inscribed octagon, which is $2\sqrt{2}$.

Prize Solution of Problem III.

By Gustavus Frankenstein, Springfield, Ohio.

III. The perpendicular to the normal of a parabola at its intersection with the principal axis, meets the curve in two points through which tangents are drawn; find the locus of their common point.

Let $x'y'$ be a point of the parabola whose equation is $y^2 = 2px$. We have always the subnormal $= p$ $\therefore$ $p + x'$ is the abscissa of the point of intersection of the normal with the axis of x (or the principal axis of the parabola), and the equation of the line drawn

through this point, perpendicular to the normal, or parallel to the tangent, is, therefore,

$$y = \frac{p}{y'}\,[x - (p + x')].$$

But $by = p(x+a)$ is the equation of the line which joins the points of contact of the two tangents to the parabola which meet in the point a, b. The identity of these equations gives $\frac{p}{y'} = \frac{p}{b}$, or $y' = b$; also, $\frac{-p(p+x')}{y'} = \frac{pa}{b}$, or $p + x' = -a$.

Substituting for x' its value $\frac{y'^2}{2p} = \frac{b^2}{2p}$, there results

$$b^2 = -2pa - 2p^2.$$

The locus is therefore another and equal parabola opening in the opposite direction, with its vertex at the distance p from that of the other.

Either parabola is the locus of the points a, b, in respect to the other parabola.

I find the same reasoning which led me to the solution of this problem, also very readily solves Problem V., No. XI., Vol. I.: — "If through any point, secants be drawn to a curve of the second order, and through the points where each secant intersects the curve, tangents be drawn meeting each other, the locus of the points of meeting is a straight line."

The equation of a line passing through the given point c, d is $y - c = A(x - d)$. The equation of the line which joins the point of contact of the two tangents to a conic section which meets in the point a, b is $2by = (m + 2an)x + am$. The identity of these equations requires that $\frac{m+2an}{2b} = A$, and $\frac{am}{2b} = c - Ad$,

$$\therefore\ 2bc = (m + 2an)\,d + am;$$

and thus

$$b = \frac{2nd + m}{2c} \cdot a + \frac{md}{2c}.$$

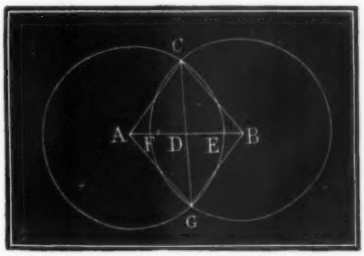
The locus of the points a, b is a straight line easily constructed.

Prize Solution of Problem IV.

By Asher B. Evans, Madison University, Hamilton, N. Y.

IV. Required the radii of three equal circles tangent to each other, and cutting off equal areas from a given circle, so that the sum of these areas may be a maximum.

Let B represent the centre and BC the radius of the given circle, A the centre and AC the radius of one of the three equal tangent circles. Let $BC = 1$, $AC = r$, area of $FCEG = u$; then will



$$AB = \tfrac{2}{3} r\sqrt{3}, \quad AD = \frac{7r^2 - d}{4r\sqrt{3}},$$

$$BD = \frac{r^2 + 3}{4r\sqrt{3}}, \quad CD = \sqrt{\frac{42r^2 - r^4 - 9}{48r^2}},$$

$$CF = \cos^{-1}\left(\frac{r^2 + 3}{4r\sqrt{3}}\right),$$

$$CE = r\cos^{-1}\left(\frac{7r^2 - 3}{4r^2\sqrt{3}}\right).$$

Area, $FCEG = ACEG + BCFG - ACBG$, or

$$(1) \quad u = r^2\cos^{-1}\left(\frac{7r^2 - 3}{4r^2\sqrt{3}}\right) + \cos^{-1}\left(\frac{r^2 + 3}{4r^2\sqrt{3}}\right) - \tfrac{1}{6}\sqrt{(42r^2 - r^4 - 9)}.$$

Differentiating (1) gives

$$(2) \quad \frac{du}{dr} = 2r\cos^{-1}\left(\frac{7r^2 - 3}{4r^2\sqrt{3}}\right) - \frac{1}{3r}\sqrt{(42r^2 - r^{\ } - 9)}.$$

Putting $\frac{du}{dr} = 0$ gives

$$(3) \quad \cos^{-1}\left(\frac{7r^2 - 3}{4r^2\sqrt{3}}\right) = \frac{2}{\sqrt{3}}\sqrt{1 - \left(\frac{7r^2 - 3}{4r^2\sqrt{3}}\right)^2}.$$

For brevity, put $\frac{7r^2 - 3}{4r^2\sqrt{3}} = x$; then (3) becomes

$$(4) \quad \cos^{-1} x = \frac{2}{\sqrt{3}}\sqrt{(1 - x^2)}.$$

This equation, solved by approximation, gives $x = .60976$, $\therefore r = 1.0385$ nearly.

Equation (2) gives

$$\frac{d^2u}{dr^2} = 2\cos^{-1}\left(\frac{7r^2-3}{4r^2\sqrt{3}}\right) - \frac{12+\frac{3}{r^2}-\frac{1}{3}r^2}{\sqrt{(42r^2-r^4-9)}},$$

which is essentially negative when $r = 1.0385$; hence r is the radius of one of the three equal circles that cut off the greatest areas from the given circle.

No correct solution of Problem V. has been received.

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN No. X., Vol. II.

The first prize is not awarded.

The second prize is awarded to A. G. Barker, Waterville College, Maine.

The third prize is awarded to William Timpson, White Plains, N. Y.

Prize Solution of Problem I.

By A. G. Barker, Waterville College, Maine.

I. Solve the equations

$$x^2y^2(x^4-y^4) = 2340, \qquad xy(xy^3-1)(x^2+y^2) = 1794,$$

by quadratics.

Performing the multiplications indicated, and adding twice the second equation to the first, there results

$$x^6y^2 + 2x^4y^4 + x^2y^6 - 2x^3y - 2xy^3 = 5928;$$

or,
$$x^2y^2(x^2+y^2)^2 - 2xy(x^2+y^2) = 5928.$$

Solving this quadratic, we get

$$xy(x^2+y^2) = 78, \quad \text{or} \quad -76.$$

Combining this result with the given equations, it is easy to find $x = 3$, $y = 2$. These were the only values of x and y found in any of the solutions.

Prize Solution of Problem II.

By William Timpson, White Plains, N. Y.

II. Given the lengths of the three perpendiculars dropped from any point in the plane of an equilateral triangle upon the sides; to find the segments of the sides.

Put $DE = a$, $DF = b$, and $DG = c$. Now,

$$\tfrac{1}{2} AB\,(DE + DF + DG) = \text{area } \triangle ABC = \tfrac{1}{2} AB \,.\, CH;$$

$$\therefore\ CH = DE + DF + DG = a + b + c.$$

Making $AH = x$, $(2x)^2 - x^2 = (a+b+c)^2$, or $2x = \frac{2}{\sqrt{3}}(a+b+c)$, the side of the triangle.

The triangles DKL, CFL, CHB are similar; and since $HB = \frac{1}{2} BC$, $FL = \frac{1}{2} LC$, and $LK = \frac{1}{2} DL$. Let $FL = y$, then $CL = 2y$, $DL = b - y$,

$$LK = \frac{b-y}{2} = a + b - 2y \ \therefore\ y = \frac{2a+b}{3},$$

and $$CL = 2y = \tfrac{2}{3}(2a + b).$$

In the same way, $NG = \frac{2a+c}{3}$, and $AN = \frac{2}{3}(2a+c)$; $PF = \frac{2c+b}{3}$, and $PB = \frac{2}{3}(2c + b)$. Now $FC = \sqrt{(CL^2 - FL^2)} = \frac{2a+b}{\sqrt{3}}$,

$$\therefore\ BF = \frac{2}{\sqrt{3}}(a + b + c) - \frac{2a+b}{\sqrt{3}} = \frac{b+2c}{\sqrt{3}}.$$

Also $AG = \sqrt{(AN^2 - NG^2)} = \frac{2a+c}{\sqrt{3}}$, $\therefore\ GB = \frac{2b+c}{\sqrt{3}}$.

And $$AE^2 + ED^2 = DG^2 + AG^2,$$

$$\therefore\ AE = \frac{a+2c}{\sqrt{3}}, \qquad \text{and} \qquad CE = \frac{a+2b}{\sqrt{3}}.$$

In solving this question, I noticed the following: —

$$AG^2 + GD^2 = AE^2 + ED^2,$$
$$BF^2 + FD^2 = BG^2 + GD^2,$$
$$CE^2 + ED^2 = CF^2 + FD^2,$$
$$\therefore AG^2 + BF^2 + CE^2 = AE^2 + BG^2 + CF^2;$$

or, $$AG^2 - GB^2 + BF^2 - CF^2 + CE^2 - AE^2 = 0;$$

or,

$$(AG+GB)(AG-GB)+(BF+CF)(BF-CF)+(CE+AE)(CE-AE)=0$$

or, $$AB(AG-GB)+BC(BF-CF)+CA(CE-AE)=0.$$

Dividing by $AB = BC = CA$,

$$AG - GB + BF - CF + CE - AE = 0,$$

or, $$AG + BF + CE = GB + CF + AE;$$

that is, the sum of the alternate segments of the sides is equal to one half the perimeter of the triangle.

Prize Solution of Problem III.

III. If e denote the edge of any regular dodecahedron, and $\beta = 36°$, prove that

$$\text{solidity} = \frac{5\,e^3 \cot^2\beta}{2\sqrt{(4\sin^2\beta - 1)}} = \frac{5\,e^3 \tan^2 45}{4\sqrt{(\sin 6° \cos 24°)}}.$$

Also obtain similar formulas for the solidity of the icosahedron.

The following solution is essentially that of Asher B. Evans, Madison University, Hamilton, N. Y.

In any of the regular polyhedrons, let

e = length of an edge.	R = radius of inscribed sphere.
F = number of faces.	r = radius circle inscribed in a face.
n = number of sides to a face.	G = area of a face.
m = number of faces at a solid angle.	V = solidity of the polyhedron.

Let C be the centre of the polyhedron, A the centre of a face, B the vertex of one of the adjacent solid angles, and E the middle point of the adjacent edge.

Let $H =$ area of triangle AEB, and $\alpha =$ angle ACE. Then the angle between the planes ACE and ACB will be $\frac{\pi}{n}$; and that between the planes ACB and BCE will be $\frac{\pi}{m}$; and the intersections of these three planes with the inscribed sphere will form a right spherical triangle, of which one side will be $R\alpha$, and the two angles will be $\frac{\pi}{n}$ and $\frac{\pi}{m}$.

Whence $$\cos \alpha = \frac{\cos \frac{\pi}{m}}{\sin \frac{\pi}{n}}.$$

In the right triangle AEB, the angle at A will be $\frac{\pi}{n}$, and $EB = \frac{1}{2} e$; whence

$$AE = r = \tfrac{1}{2} e \cot \frac{\pi}{n}, \qquad H = \tfrac{1}{8} e^2 \cot \frac{\pi}{n},$$

$$G = 2nH = \tfrac{1}{4} e^2 n \cot \frac{\pi}{n},$$

$$R = r \cot \alpha = \frac{\frac{1}{2} e \cot \frac{\pi}{n}}{\sqrt{\left(\frac{\sin^2 \frac{\pi}{n}}{\cos^2 \frac{\pi}{m}} - 1\right)}},$$

$$V = F \times G \times \tfrac{1}{3} R = \frac{e^3 F n \cot^2 \frac{\pi}{n}}{24 \sqrt{\left(\frac{\sin^2 \frac{\pi}{n}}{\cos^2 \frac{\pi}{m}} - 1\right)}}.$$

For the

Tetrahedron, $F = 4,\ n = 3,\ m = 3,\ V = \frac{e^3 \sqrt{2}}{12}$.

Cube, $F = 6,\ n = 4,\ m = 3,\ V = e^3$.

Octahedron, $F = 8,\ n = 3,\ m = 4,\ V = \frac{e^3 \sqrt{2}}{3}$.

Dodecahedron, $F = 12,\ n = 5,\ m = 3,\ V = \frac{5 e^3 \cot^2 36°}{2 \sqrt{(4 \sin^2 36° - 1)}}$

$$= 5 e^3 \sqrt{\left(\frac{47 + 21\sqrt{5}}{40}\right)} = \frac{5 e^3 \tan 45°}{4 \sqrt{(\sin 6° \cos 24°)}}.$$

Icosahedron, $F = 20,\ n = 3, m = 5,\ V = \frac{5 e^3 \cot^2 60°}{2 \sqrt{\left(\frac{\sin^2 60°}{\cos^2 36°} - 1\right)}} = 5 e^3 \left(\frac{3 + \sqrt{5}}{12}\right)$.

Mr. EVANS's solution of Problem IV. is vitiated by a slight oversight in the last case.

No correct solution of Problem V. has been received, probably owing to a misapprehension of the problem. The bisection referred to in the problem is general, and independent of any law of variation in density. Hence a plane cannot be the bisecting surface.

SIMON NEWCOMB,
W. P. G. BARTLETT,
TRUMAN HENRY SAFFORD.

DISCUSSION OF THE EQUATIONS WHICH DETERMINE THE POSITION OF A COMET OR OTHER PLANETARY BODY FROM THREE OBSERVATIONS.

By G. W. HILL, Nyack Turnpike, N. Y.

THESE equations may be found in the *Mécanique Céleste*, Vol. I., p. 207, or in PONTECOULANT, *Théorie Analytique du Système du Monde*, Vol. II., p. 21, where the analysis conducting to them is given. The equations are

$$\varrho = \frac{R^4}{m}\left(\frac{1}{r^3} - \frac{1}{R^3}\right), \qquad r^2 = R^2 - 2R\varrho\cos c + \varrho^2;$$

in which ϱ is the comet's distance from the earth, r its radius vector, R the earth's radius vector, c the angular distance of the comet from the sun, and m a function of known quantities. The unknown quantities are ϱ and r. Now, to find the values of ϱ and r, it has always been advised to determine them from the equations in the above form by the method of double position. But this way is needlessly complex in calculation, and we shall give the equations a much simpler form, to which the tentative process can be easily applied. Assume $r = \frac{R\sin c}{\sin\theta}$, and $\varrho = \frac{R\sin(\theta - c)}{\sin\theta}$; these values substi-

tuted in the last equation render it identical; and from the first we obtain

$$\frac{\sin^3 \theta}{\sin^3 c} = 1 + m \frac{\sin (\theta - c)}{\sin \theta};$$

or, $$\sin^4 \theta = \sin^3 c \,[(1 + m \cos c) \sin \theta - m \sin c \cos \theta].$$

Let $\sin^3 c\,(1 + m \cos c) = A \cos B$, and $m \sin^4 c = A \sin B$; whence

$$\tan \beta = \frac{m \sin c}{1 + m \cos c}, \quad \text{and} \quad A = \frac{m \sin^4 c}{\sin \beta} = \frac{\sin^4 c}{\sin (c - \beta)}.$$

Then the equation becomes

$$\sin^4 \theta = A \sin (\theta - \beta).*$$

This is probably the most elegant form to which the above equations can be reduced. Taking the logarithms of both sides, we have

$$4 \log \sin \theta - \log \sin (\theta - \beta) = \log A.$$

From this a near approximate value of θ can be found by inspection of the table of logarithmic sines, without any calculation but that which can be performed mentally. The exact value may then be obtained by a single application of the method of double position.

We proceed to notice some properties of the roots of the equation to which we have reduced the two first given.

The equation may be put under this form:

$$\sin^8 \theta - 2 A \cos \beta \sin^5 \theta + A^2 \sin^2 \theta - A^2 \sin^2 \beta = 0.$$

From the powers of $\sin \theta$, which are wanting, we perceive that the equation has at least four imaginary roots; and the sign of the last term being negative, there are at least two real roots, one positive, the other negative. By reference to the equations which determine ϱ and r, it will be seen that they are satisfied by the values $\varrho = 0$,

* See Gauss's *Theoria Motus*, and especially the Appendix to Commander Davis's translation. — Ed.

and $r = R$; the corresponding value of θ is c, which, put in the equation $\sin^4 \theta = A \sin(\theta - \beta)$, renders it identical. Since, by the nature of the problem only positive values of ρ and r are admissible, and consequently θ being contained between the limits c and π, we may reject as useless the four imaginary roots, and the negative root and the root $\theta = c$. There remain two roots which are necessarily real, because the problem must have at least one solution, and we are led to the important conclusion that it cannot have more than two.

Let us here deduce a relation which exists between the known quantities when these two roots are real. Differentiating the equation $4 \log \sin \theta - \log \sin(\theta - \beta) = \log A$ with respect to θ, we get for the equation containing the values of θ, when

$$4 \log \sin \theta - \log \sin(\theta - \beta)$$

is a maximum or minimum,

$$4 \cot \theta - \cot(\theta - \beta) = 0, \quad \text{or} \quad 4 \tan(\theta - \beta) = \tan \theta;$$

from which we derive

$$\tan \theta = \frac{3 \pm \sqrt{(9 - 16 \tan^2 \beta)}}{2 \tan \beta}.$$

If $\tan^2 \beta > \frac{9}{16}$, this value of $\tan \theta$ is imaginary, and the expression $4 \log \sin \theta - \log \sin(\theta - \beta)$ is not susceptible of a maximum or minimum value, and the equation in $\sin \theta$ has only two real roots, which are among those rejected. Hence we conclude, that when the data are taken from observation, the quantity $\frac{m \sin c}{1 + m \cos c}$ will always be contained between the limits $\pm \frac{3}{4}$. If we substitute 0 for θ in the equation $\sin^4 \theta - A \sin(\theta - \beta) = 0$, the result is $A \sin \beta$, and for $\theta = \pi$ the result is $-A \sin \beta$; showing the existence of an odd number of roots between the limits $\theta = 0$ and $\theta = \pi$, which odd number is three, since c and the root which the

problem demands are within these limits. If we make $\theta = c + dc$, there results the quantity $(4 \sin^3 c \cos c - A \cos(c - \beta))\, dc$; or,

since $\quad A = \frac{\sin^4 c}{\sin(c-\beta)}, \qquad \tan\beta = \frac{m \sin c}{1 + m \cos c},$

the quantity $(3 \cos c - m) \sin^3 c\, dc$. And $-A \sin\beta$, the result on putting $\theta = \pi$, is equal to $-m \sin^4 c$. Therefore, $\sin c$ being always positive, θ has two real values, or only one (between the limits c and π), and consequently the problem two or one answer, according as m and $m - 3 \cos c$ have the same or different signs.

It is evident that $A \cos\beta$ must be positive, in order that the equation in $\sin\theta$ may have three real positive roots; so the quantity $1 + m \cos c$ is always positive, and $\tan\beta$ has the same sign as m. If β be taken between the limits $\pm \frac{\pi}{2}$, A is always positive. Since the equation in $\sin\theta$ must have no root greater than one, units substituted for $\sin\theta$ in the first derived function of its equation must render it positive; that is, the expression $4 - 5A \cos\beta + A^2$ is positive, which gives $A < 2$, and $A \cos\beta < \frac{4}{5}$. According as m is positive or negative, the equation for finding θ presents itself under two shapes, $\sin^4\theta = A \sin(\theta - \beta)$, and $\sin^4\theta = A \sin(\theta + \beta)$, in which A is always positve and less than 2, and β never exceeds $36° 53'$.

From the expression for ρ in terms of r, it is clear that r is less or greater than R, according as m is positive or negative. Therefore, in the first case, θ is contained between the limits c and $\pi - c$; and in the second case, if c is in the first quadrant, between $\pi - c$ and $\pi - \beta$; but if c be in the second quadrant, between c and $\pi - \beta$. These remarks may be of use to shorten the tentative process of finding θ.

With regard to θ, it is clear it is the angle subtended at the comet by its radius vector and the line joining it and the earth prolonged beyond the comet.

Mathematical Monthly Notices.

Elements of Chemical Physics. By JOSIAH P. COOKE, JR., Erving Professor of Chemistry and Mineralogy, in Harvard University. Boston: Little, Brown, & Co. 1860. 8vo. pp. 739.

This is the first of an expected series of three volumes, to form "an extended work on the Philosophy of Chemistry." It is, however, in itself a complete treatise of the principles involved in the processes of weighing and measuring.

The introduction gives a view of the nature of physical and chemical laws as seen by the light of a metaphysical theory of causation, rather doubtful in itself, but very popular among modern theologians.

In the second chapter, under the head of "General Properties of Matter," we have an account of the English and French measures of length, and the elementary principles of mechanics, including some formulæ for the motion of the pendulum. From some of the reasoning and conclusions respecting *vis-viva* in this chapter, we are disposed to dissent, as being inaccurate, and the term "centre of gravity" seems to be sometimes used in a very unusual sense.

Chapter III., entitled "The Three States of Matter," gives a highly interesting, extended, and luminous account of the characteristic properties of solids, liquids, and gases, with many valuable tables. Upwards of sixty pages of this chapter are devoted to crystallography, profusely illustrated by diagrams on a black ground; and eighty pages to the mutual molecular actions between matter in its three forms, taken two and two: e. g. between solids and solids, solids and liquids, etc.

The fourth chapter is devoted to the *chemical* theory of heat; and seems to be much more full and satisfactory than the chapters on that subject in any work previously published in this country. It includes accounts of the results of REGNAULT and DULONG on the specific heat and expansion of gases, of PERSON'S theory of the absolute zero of heat, and of JOULE'S experiments on the mechanical theory of heat, so interesting to the physical philosophers; all the results being deduced mathematically, where necessary, and expressed quantitatively, where it is possible. It contains a vast and patiently collected mass of information.

As a general text-book for the student, it labors under the disadvantage of not completely filling the course of either physics or chemistry, as usually taught in colleges; but we think the third and fourth chapters might with some advantage, and a great deal of pleasure to the student, be substituted for so much of the course of chemistry as treats of the properties of the -yles, -ines, -ones, etc. The style of these chapters is so free, the illustrations so excellent and numerous, and the typography so good, that no one possessed of the slightest taste for physical science could fail to be interested by them. To the special student in chemistry the work must be both interesting and valuable, from its exhaustive character.

Elements of Analytical Geometry and of the Differential and Integral Calculus. By CHARLES DAVIES, LL.D., Professor of Higher Mathematics, Columbia College. 2 vols. in one. New York: A. S. Barnes and Burr. 1860. pp. 204, 194.

That there is a growing interest in the subjects treated in these volumes there can be no doubt. Their study has been gradually working its way into seminaries, academies, and even into many high-schools. In these institutions a large proportion of students end their course of study, and for such the work before us is mainly intended. In view of such a use, the manner of presenting and the arrangement of the propositions, and especially what and how much to retain out of such abundance, become very serious questions. In these respects,

as well as in the clearness and simplicity with which the work is written, the author has been remarkably successful. The discussion of the general equation of the second degree is brief, but very clear and satisfactory. The arrangement is as follows: — Introduction. Book I. Points and Lines in a Plane. Books II. – V. Of the Circle, Ellipse, Parabola, Hyperbola. Book VI. Space, Point, and Line, Surfaces of the First and Second Orders. It is enough to say of this part of the volume, that it is a judicious abridgment of the Author's Analytical Geometry.

But Part Second, to which we wish to call more particular attention, shows even more marked evidence of thought and care in arrangement and detail. With the arrangement we are especially pleased, and believe it will receive the approbation of all practical teachers. In Section I. we find Definitions and First Principles; II. Differentials of Algebraic Functions; III. Integration and Applications; IV. Successive Differentials, &c.; V. Maxima and Minima; VI. Differentials of Transcendental Functions; VII. Transcendental Curves; and, lastly, Integration of Rational Fractions, Integration by Parts, and a Discussion of the Cases in which Binomials can be integrated. It will be seen that the processes of differentiation and integration are combined, instead of the usual division. The integration of the various classes of functions immediately follows their differentiation, and then the related application of both processes. The advantages of this combination of the two converse processes are obvious; and especially when the student has but little time to devote to the subject.

We sincerely regret that we cannot give the same hearty assent to the author's exposition of the first principles of the subject. For brevity, and to be sure of presenting his theory completely, we quote, retaining his italics. "Where no law of change has been fixed, such a law may be imposed as will make the variable to change uniformly, and to pass through all values between any two limits. There exists a constant difference between two consecutive values, which forms no appreciable part of the quantity itself." — *Introduction.*

"If the change is *uniform*, and such as to embrace *every value* between given limits, such change cannot be expressed by a number whose unit is any part of 1. For, however small the decimal denoting the change, it might still be diminished by prefixing ciphers; hence, contrary to the supposition, there would be values which h did not pass through. Thus we cannot express the change by numbers, and a new language becomes necessary." § II.

"Before N can coincide with M, there must be a *last* value of $x' - x$; we denote it by dx; and x, $x + dx$, are called *consecutive*. *Consecutive values admit of no intermediate value under the same law of change*; hence their difference cannot be expressed by a number, but is, *numerically* $= 0$. It is named a *differential*." § 13.

"dx is smaller than any number; its numerical value is 0. $(dx)^2$ is 0, compared with dx; i. e. infinitely less than dx." § 20.

We infer (1) that *before* $x' - x$ *vanishes*, it has a "last value" which really exists, but which is too small to be further diminished by prefixing ciphers; and that all so-called continuous change is actually composed of an infinity of such steps; (2) that while dx is such a *minimum possibile*, yet dy is smaller if the derivative $\frac{dy}{dx}$ be between -1 and $+1$; and that $(dx)^2$ is infinitely less than dx. This is the meaning, unless the proviso "under the same law of change" modifies it in some way which we do not clearly understand, and which may well perplex the young student for whom the book is written. Now (1) is manifestly untrue; it is also incompatible with (2). But were both (1) and (2) true, we think two objections to the theory would remain. First, it involves the mystery of continuity, and also the inconceivable idea of *absolute* infinitesimals or infinites of different orders; while the calculus requires us to discuss neither. Second, it treats differentials as magnitudes *essentially fixed*, while the spirit of the Calculus is to regard them as *essentially arbitrary*. We prefer to define them as any changes

taken so small that the character of the result could not be changed by taking them still smaller; and derivatives as the *limits* toward which $\frac{dy}{dx}$, &c. finally *approach*, as dx approaches zero.

The author demonstrates the theorems of MACLAURIN and TAYLOR separately, and gives no hint that the two are identical. But by making $a = 0$, TAYLOR'S development of $f(a+x)$ in powers of x, becomes MACLAURIN'S development of $f(x)$; while putting $\phi(x) = f(a+x)$, MACLAURIN'S development of $\phi(x)$ becomes TAYLOR'S development of $f(a+x)$.

We have thus, as fully as space will permit, pointed out what we consider as defects of, in other respects, an admirable work; one having too many positive merits not to be improved by just adverse criticism.

Elements of Geometry, with Practical Applications to Mensuration. By BENJAMIN GREENLEAF, A. M., Author of "The National Arithmetic," "Treatise on Algebra," &c. Fourth Electrotype Edition. Boston: Published by Robert S. Davis & Co. 1859.

This work is the last of the series yet published, beginning with the "National Arithmetic," which was first issued in 1835. We understand that a work on Plane and Spherical Trigonometry is soon to be added. The "National Arithmetic," by frequent revisions, has been kept fully up with the times. The last revision, in 1857, has left it an excellent work, especially for fitting students for business life, and its clearly expressed rules will always make it a good book for reference, as well as text-book.

The Geometry is based on that of LEGENDRE, and for the sake of comparison we append the table of Contents. Book I. First Principles. II. Ratio and Proportion. III. The Circle, and the Measure of Angles. IV. Proportions, Areas, and Similarity of Figures. V. Problems Relating to the preceding Books. VI. Regular Polygons, Area of Circles. VII. Planes, Diedral and Polyhedral Angles. VIII. Polyhedrons. IX. The Sphere and its Properties. X. The Three Round Bodies. XI. Applications of Geometry to the Mensuration of Plane Figures. XII. Applications to Solids. XIII. Miscellaneous Geometrical Exercises. XIV. Application of Algebra to Geometry. This will show to what extent the excellent work of LEGENDRE has been reproduced in the one before us. In the different books we notice many changes in the order of the propositions, and a few in the methods of demonstration; and many of them appear to us as the result of experience in the school-room, and as better adapting the work to the wants of beginners. But in our opinion its crowning excellence is the clearness and conciseness with which the definitions and demonstrations are expressed. The greatest care has evidently been taken with the style; and after a careful search we have not been able to find one expression which we felt sure could be improved. Book XIII., comprising undemonstrated propositions, and problems for construction and numerical solution, is another valuable feature, and the only fault is that they are too few. For instance, only ten of the undemonstrated propositions relate to Book I., when there ought to have been at least thirty-five, one to each of the demonstrated propositions. Besides, these propositions and problems would be more likely to be used if they had been inserted in their proper places throughout the book, just what should have been done with Books XI. – XIX. A book on Arithmetic or Algebra would not be tolerated with so few exercises, and all put at the end. There are advantages in not bringing in the definitions before they are wanted. If the definitions of parallels had been based on the idea of direction, as that of an angle is, the change from LEGENDRE would, in our opinion, have simplified several of the earlier propositions much more than that adopted by the author has done. The changes we have suggested are matters of opinion, and by no means imply that the work is not one which we can heartily recommend to the attention of teachers. Indeed, we believe that teachers will generally introduce all the exercises and applications in their proper places, and thus remove our only objection to the arrangement.

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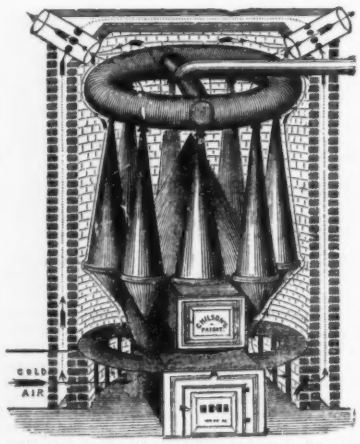

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